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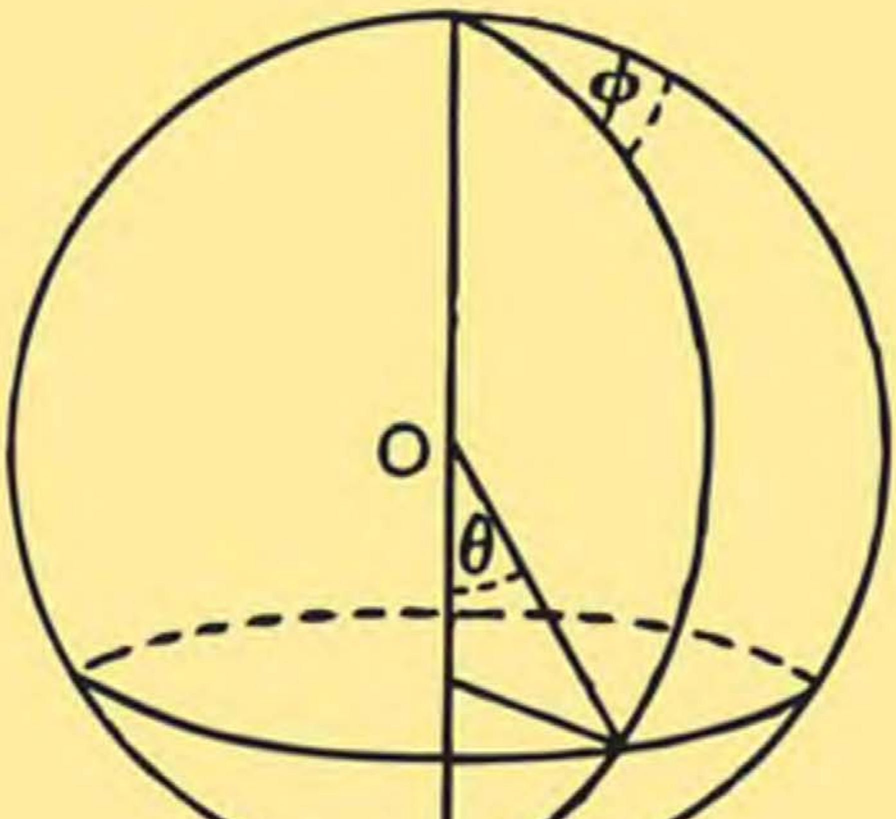
DYNAMICS

A Text-Book for the use of the Higher Divisions in Schools and for First Year Students at the Universities

Arthur Stanley Ramsey



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Dynamics

A.S. Ramsey (1867-1954) was a distinguished Cambridge mathematician and President of Magdalene College. He wrote several textbooks 'for the use of higher divisions in schools and for first-year students at university'. This book on dynamics, published in 1929, was based upon his lectures to students of the mathematical tripos, and reflects the way in which this branch of mathematics had expanded in the first three decades of the twentieth century. It assumes some knowledge of elementary dynamics, and contains an extensive collection of examples for solution, taken from scholarship and examination papers of the period. The subjects covered include vectors, rectilinear motion, harmonic motion, motion under constraint, impulsive motion, moments of inertia and motion of a rigid body. Ramsey published a companion volume, Statics, in 1934.

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Dynamics

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ARTHUR STANLEY RAMSEY

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A Text-Book for the use of the Higher Divisions in Schools and for First Year Students at the Universities

by

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PREFACE

This book is intended primarily for the use of students in the higher divisions in schools, particularly for those who intend to take an Honours Course of Mathematics at a University, and also for University students preparing for a first Honours Examination. It is based upon courses of lectures given during many years to first-year students preparing for the Mathematical Tripos, and it is assumed that the majority of readers will already have acquired some knowledge of elementary dynamics. Although the book contains chapters on Orbits and the dynamics of Rigid Bodies, none the less it may claim to be a text-book on *Elementary* Dynamics, for there is probably no branch of elementary Mathematics the content of which has expanded so greatly in the last twenty years.

One of the changes that accompanied the reform of the Mathematical Tripos was the removal of the restriction that Elementary Mechanics meant Mechanics without the Calculus. This restriction set well-defined and narrow bounds to the subject and the new regulations which gave teachers and students freedom to use any analytical methods in their work have been far reaching in their effect. Though the schedule in Dynamics for Part I of the new Tripos has remained unaltered, successive Examiners have added considerably to the interpretation of its contents. To give one instance only—the phrase 'motion under gravity' is now understood to mean 'in a resisting medium'—and it would be easy to give other examples of the elasticity of interpretation to which the schedule lends itself. The result of this change is that a first-year course in Dynamics at the University now includes all the easier problems of two-dimensional dynamics stopping short of the use of moving axes and Lagrange's Equations. This growth in the content of Elementary Dynamics has been a gradual process and undoubtedly beneficial to the study of the subject and stimulating to the average student. It is inevitable that its effect will extend to the schools, if it has not already done so; and it is not unreasonable to suppose that before many years have passed, candidates for Scholarships in Mathematics will be expected to possess a wider knowledge of dynamics embracing such parts of the subject as 'motion under simple central forces' and the elements of uniplanar rigid dynamics. The object of this book is to assist in this development. It is hoped that the presentation of the subject will prove sufficiently simple. An attempt has been made to preserve the conciseness of lecture notes and at the same time to give detailed explanations where experience has shewn that students find difficulties. Besides examples for solution the book contains a large number of worked examples; some of these are of purpose very simple illustrations of the theory, while others are of a more difficult kind for the assistance of readers who wish to learn how to work harder examples. The examples are nearly all taken from Scholarship papers or Tripos papers and the source is indicated by the letters S. and M. T. No attempt is made to exhaust the subject and the later chapters are only intended to be suggestive of the kinds of problems that can be solved, without elaborate analysis, as examples of the fundamental theorems; some few of these may prove to be too difficult for weaker students and they are intended rather to introduce abler students to more advanced work.

In conclusion I desire to express my thanks to the printers and readers of the University Press for their excellent work in the setting up of the book and the elimination of mistakes, and also to say that if the book contains errors I shall be grateful to anyone who will point them out.

A. S. RAMSEY

30 *Nov.* 1928
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DYNAMICS

Chapter I

INTRODUCTION

1·1. The subject of Dynamics is generally divided into two branches: the first, called Kinematics, is concerned with the geometry of motion apart from all considerations of force, mass or energy; the second, called Kinetics, is concerned with the effects of forces on the motion of bodies.

1·2. In order to describe the motion of a body or of a point two things are needed, (i) a frame of reference, (ii) a time-keeper. It is not possible to describe absolute motion, but only motion relative to surrounding objects; and a suitable frame of reference depends on the kind of motion that it is desired to describe. Thus if the motion is rectilinear the distance from a fixed point on the line is a sufficient description of the position of the moving point; and in more general cases systems of two or of three rectangular axes may be chosen as a frame of reference. For example, in the case of a body projected from the surface of the Earth a set of axes with the origin at the point of projection would be suitable for the description of motion relative to the Earth. But, for the description of the motion of the planets, it would be more convenient to take a frame of axes with an origin at the Sun's centre.

1·3. It is important to realize that there is no such thing as absolute time, but the period of rotation of the Earth relative to the fixed stars provides a unit of time, *the sidereal day*, which, so far as it can be tested with other time measures, is constant and therefore adequate for the purposes of ordinary dynamics.

1·4. The functions involved in dynamical problems are for the most part differential coefficients with regard to 'time,' 't,' as the independent variable. Thus 'motion' is 'change of position' or 'displacement,' 'velocity' is 'rate of displacement' and 'acceleration' is 'rate of change of velocity.' Hence, if x denotes a distance, dx/dt denotes a velocity and d^2x/dt^2 denotes an acceleration. The formulation of a dynamical problem therefore in general consists of one or more relations between certain variables (coordinates of position) and their differential coefficients with regard to time. Such relations are called differential equations.

NOTE ON DIFFERENTIAL EQUATIONS

1·5. It is assumed that the reader is acquainted with the elementary processes of differentiation and integration.

A *differential equation* is a relation between an independent variable t, a dependent variable x, and one or more of the differential coefficients of x with regard to t. The *order* of a differential equation is that of the highest differential coefficient that it contains. A *solution* of a differential equation is a relation between x and t that satisfies the equation, and the *complete solution* of a differential equation is a relation between x, t and one or more arbitrary constants of integration, the number of such constants being equal to the order of the equation.

For example:

$$\text{(i)}\quad \frac{dx}{dt}-2x=0$$

is a differential equation of the first order. It will be found on substitution that $x=e^{2t}$ is a solution; and the complete solution is $x=Ce^{2t}$, where C is an arbitrary constant.

$$\text{(ii)}\quad \frac{d^2x}{dt^2}+x=0$$

is a differential equation of the second order. It has solutions

$$x=\sin t \quad\text{and}\quad x=\cos t,$$

and the complete solution is

$$x=A\sin t+B\cos t,$$

where A and B are arbitrary constants.

1·6. The differential equations of dynamics are of either the first or second order.

Equations of the First Order.

We may have to deal with equations in which the variables can be separated. Such equations can be put in the form

$$M\,dx/dt=N \quad\text{..............................(1)},$$

where M is a function of x only (or a constant) and N is a function of t only (or a constant). The complete solution is

$$\int M\,dx=\int N\,dt+C \qquad\text{.........................(2),}$$

where C is an arbitrary constant.

For example, the equation

$$x\frac{dx}{dt}=g-kx^2$$

is solved by writing

$$\frac{x\,dx}{g-kx^2}=g\,dt,$$

so that

$$-\frac{1}{2k}\log(g-kx^2)=gt+C$$

is the complete solution.

1·61. Another type of equation that sometimes occurs in dynamics is the *linear equation of the first order*. A differential equation is said to be linear when it does not contain powers or products of the dependent variable x and its differential coefficients. Thus the linear equation of the first order is

$$dx/dt+Mx=N \qquad\text{.............................(3),}$$

where M, N are functions of t or constants.

The solution is effected by first multiplying both sides of the equation by $e^{\int M dt}$ and then integrating; because it can easily be verified that

$$\frac{d}{dt}(xe^{\int M dt})=\left(\frac{dx}{dt}+Mx\right)e^{\int M dt}$$

Hence $$xe^{\int M dt}=\int e^{\int M dt}N\,dt+C \qquad\text{..........................(4),}$$

where C is an arbitrary constant.

We note that if M is a constant the solution is

$$xe^{Mt}=\int e^{Mt}N\,dt+C \qquad\text{..........................(5).}$$

For example, the equation

$$\frac{dx}{dt}+kx=gt$$

can be integrated if both sides are multiplied by e^{kt}, giving on integration

$$xe^{kt}=g\int e^{kt}t\,dt+C$$

$$=ge^{kt}\left(\frac{t}{k}-\frac{1}{k^2}\right)+C,$$

or $$x=\frac{g}{k}\left(t-\frac{1}{k}\right)+Ce^{-kt} \qquad\text{..........................(6).}$$

1·7. Equations of the Second Order.

A common type of differential equation of the second order is

$$\frac{d^2x}{dt^2}+2a\frac{dx}{dt}+bx=0 \qquad\text{..........................(7),}$$

where a and b are constants.